Thermomagnetic Force for Small Spheroidal Particles*

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The thermomagnetic force for small spheroidal particles is given in the Waldmann limit. In this limit the force is proportional to the surface area of the particle, and depends on the gas pressure p and on the magnetic field strength H only via H/p. For the gases HD, CH_4, CO and N_2 the ratio of the thermomagnetic and thermal forces is calculated from Senftleben-Beenakker effect data. It is pointed out that the sign and magnitude of the thermomagnetic force in the Waldmann limit give information on the relative sign of two non-diagonal collision cross sections. The dependence of the force on the orientations of the magnetic field and of the axis of the particle relative to the temperature gradient is discussed.

Measurements of the magnetic field induced change in the thermal force on a thin disk suspended in a polyatomic gas were first made by Taboada [1] and reported in 1970 by Larchez and Adair [2] for O_2 and later aalso for NO [3]. Recently, this thermomagnetic force (TMF) on a disk has been studied by Taboada [4] for the gases N_2 and CO, and by Davis [5] for N_2 and HD.

These experiments [1–5] show a close relationship between the TMF and the Senftleben-Beenakker effect [6], in particular to the field effect of thermal conductivity. But, typically, the TMF is an effect which only occurs in a rarefied polyatomic gas in a magnetic field. The same criteria apply to the thermomagnetic torque, discovered by Scott et al. [7], and to the thermomagnetic pressure difference studied by Hulsman et al. [8] and Eggermont et al. [9]. Theoretical descriptions of these two effects are available for the near continuum regime: Waldmann [10] emphasized the importance of slip effects ("thermomagnetic slip") for an explanation of the thermomagnetic torque, whereas Levi et al. [11] considered the contributions arising from second order derivatives of temperature ("Maxwell stresses"). The existence of thermomagnetic slip leads to "thermomagnetic pressure differences", as predicted by Waldmann [10]. The theory has been worked out in more detail by Vestner [12].

The first calculations of the TMF for spheres have been done by Hess [13] for two pressure regions: (1) in the near continuum regime and (2) in the Waldmann limit. In case (1) the TMF (as well as the thermal force) is proportional to the radius of the particle. The TMF dependence on the strength of the magnetic field H is (in the simplest approximation) given by that of the viscosity coefficients, and its magnitude is inversely proportional to the gas pressure p (for fixed H/p). The Waldmann limit applies whenever the mean free path of a gas molecule is much larger than the particle, but is still small on a "macroscopic scale", e. g. much smaller than the distance between plates creating the temperature gradient. Then, the TMF is proportional to the surface area of the particle, and (for fixed H/p) independent of pressure. The magnetic field dependence is entirely determined [13] by that of the heat conductivity coefficients. In particular, the TMF depends on the strength H of the magnetic field and of pressure p only in the combination H/p. The basic physical processes which determine the force are slip phenomena in case (1), and the direct momentum transfer from the gas to the particle in case (2), respectively.

This paper deals with the thermomagnetic force on spheroidal particles in the Waldmann limit. In the first section, the general relation between the force and the translational heat flux, as derived earlier [14], is stated. The method applied for the derivation of this relation [14] is essentially the same as used by Waldmann [15] for the treatment of the frictional, thermal, and diffusiophoretic forces on spheres. The second section is devoted to the calculation of the relative magnitude of the field induced change in the translational heat flux from

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data available from the Senftleben-Beenakker effect of viscosity and of thermal conductivity. Numerical results are presented for the gases HD, CH_4, CO and N_2. It is shown that the measurement of the TMF in the Waldmann limit provides information on the relative sign of two non-diagonal collision cross sections. The general formula for the TMF is given in Section 3, and the special expressions for disks, spheres and cylinders are stated. Finally, in the last section, the dependence of the force on the mutual orientations of the magnetic field, the axis of the particles, and the temperature gradient is considered for a few special cases.

1. Relation Between the Force and the Heat Flux

Consider a particle which is much smaller than the mean free path l of a gas molecule, where the presence of the particle will not disturb the distribution function of the approaching molecules. The force on a suspended body can be calculated if the distribution of the molecules leaving its surface is specified. This method has been used by Epstein [16], Einstein [17], and Waldmann [15] to derive expressions for the frictional [16, 15] the thermal [17, 15] and the diffusiophoretic [15] forces. For the thermal force, in particular, the hydrodynamic expansion for the distribution function of the heat conducting gas has been inserted, i. e. the mean free path l is assumed to be much smaller than the size of a macroscopic container. In this limit the thermomagnetic force for a sphere has been given by Hess [13].

A body suspended in a heat conducting polyatomic gas is subject to the force [14]

$$\boldsymbol{K} = \tfrac{2}{5}\,(\pi\,\bar{c})^{-1} \iint \mathrm{d}\sigma\,(\boldsymbol{n}\,\boldsymbol{n} + \boldsymbol{\delta}) \cdot \boldsymbol{q}_{\text{trans}}\,. \quad (1.1)$$

Here, $\boldsymbol{q}_{\text{trans}}$ is the translational heat flux in the gas, and $\bar{c} = (8\,k\,T_0/\pi\,m)^{1/2}$, where T_0 is the mean gas temperature, m is the molecular mass, k is the Boltzmann constant; $\boldsymbol{\delta}$ is the second rank unit tensor, and $\boldsymbol{n}$ denotes the outward pointing unit normal of the surface element $\mathrm{d}\sigma$ of the particle. Eq. (1.1) has been derived [14] under the assumption of complete diffuse reflection.

For a particle of ellipsoidal shape we have

$$\frac{1}{F} \iint \mathrm{d}\sigma\,\boldsymbol{n}\,\boldsymbol{n} = N\,\boldsymbol{\delta} + (1 - 3\,N)\,\boldsymbol{s}\,\boldsymbol{s}\,, \quad (1.2)$$

hence the force can be rewritten as

$$\boldsymbol{K} = \tfrac{2}{5}(\pi\,\bar{c})^{-1}\,F[2\,(1-N)\,\boldsymbol{s}\,\boldsymbol{s} + (1-N)\,(\boldsymbol{\delta} - \boldsymbol{s}\,\boldsymbol{s})] \cdot \boldsymbol{q}_{\text{trans}}\,. \quad (1.3)$$

The surface F and the quantity N depend on the semi-axes $a_{\|}$ and $a_{\perp}$ parallel and perpendicular to the symmetry axis $\boldsymbol{s}$ of the ellipsoid, respectively [14]. Here F and N are stated for a sphere of radius a

$$F = 4\,\pi\,a^2\,, \quad N = 1/3\,, \quad (1.4)$$

a thin disk of radius $a_{\perp}$ and thickness $2\,a_{\|}$ $(a_{\|}/a_{\perp} \to 0)$

$$F = 2\,\pi\,a_{\perp}^2\,, \quad N \to 0\,, \quad (1.5)$$

and a rod of length $2\,a_{\|}$ and radius $a_{\perp}$ $(a_{\perp}/a_{\|} \to 0)$

$$F = \pi^2\,a_{\perp}\,a_{\|}\,, \quad N \to 1/2\,. \quad (1.6)$$

For a circular cylinder of length L and radius a Eq. (1.2) applies also, with

$$F = 2\,\pi\,a\,L(1 + a/L)\,, \quad N = \tfrac{1}{2}\,(1 + a/L)^{-1}\,, \quad (1.7)$$

i. e. a rod experiences the same force as a thin cylinder $(a = a_{\perp})$ of length $L = a_{\|}\,\pi/2$ (with $a/L \to 0$).

2. The Translational Heat Flux

According to Eqs. (1.1) and (1.3) the thermal force is determined by the translational heat flux [15]. In particular, the thermomagnetic force is characterized by the field dependence of the heat conductivity coefficients [13]. In the presence of a magnetic field $\boldsymbol{H} = H\,\boldsymbol{h}\,(\boldsymbol{h} \cdot \boldsymbol{h} = 1)$ the translational heat flux can be written in the form [18]

$$\boldsymbol{q}_{\text{trans}} = -\lambda^{\text{trans}}[\nabla T + F_{\|}\,\boldsymbol{h}\,\boldsymbol{h} \cdot \nabla T + F_{\perp}\,(\boldsymbol{\delta} - \boldsymbol{h}\,\boldsymbol{h}) \cdot \nabla T + F_{\text{tr}}\,\boldsymbol{h} \times \nabla T]\,. \quad (2.1)$$

The quantities $F_{\|}$, $F_{\perp}$, F_{tr} are functions of the field strength H which vanish for $H = 0$, i. e.

$$\boldsymbol{q}_{\text{trans}}(\boldsymbol{H} = \boldsymbol{O}) = -\lambda^{\text{trans}}\,\nabla T\,. \quad (2.2)$$

The field dependence of the translational heat flux is due to its collisional coupling with field dependent polarizations. Measurements of the Senftleben-Beenakker effect [6] indicate that the so called Kagan-Vector $\boldsymbol{B} \propto \langle \boldsymbol{c} \cdot \overline{\boldsymbol{J}\,\boldsymbol{J}} \rangle$ is the dominant type of polarization present in a heat conducting polyatomic gas. Here, $\boldsymbol{c}$ is the molecular velocity, $\boldsymbol{J}$ the rotational angular momentum, and $\langle \ldots \rangle$ denotes a nonequilibrium average. Then, the functions $F_{\|}$, $F_{\perp}$, F_{tr} are given by [19]

$$F_{\parallel} = -b\, f(\varphi_b)\,,\; F_{\perp} = -\tfrac{1}{2} b[f(\varphi_b) + 2 f(2\varphi_b)]\,,$$
$$F_{tr} = -\tfrac{1}{2} b[g(\varphi_b) + 2 g(2\varphi_b)]\,, \tag{2.3}$$
with $f(\varphi_b) = \dfrac{\varphi_b^2}{(1+\varphi_b^2)}\,,\quad g(\varphi_b) = \dfrac{\varphi_b}{(1+\varphi_b^2)}\,.$

The precession frequency of a molecule with a gyromagnetic ratio γ in a magnetic field of strength H is $\omega_H = \gamma H$. Consequently, $\varphi_b = \omega_H/\omega_b$ is the angle over which the magnetic moment precesses about the direction $\boldsymbol{h}$ of the magnetic field during a time of free flight ω_b^{-1}. Precisely, ω_b is the relaxation constant of the Kagan vector; it is proportional to the mean gas pressure. Hence, $\varphi_b \propto \gamma H/p_0$, i. e. the translational heat flux and the thermomagnetic force depend on the field strength H and on pressure p_0 only in the combination H/p_0. In saturation where $|\varphi_b| \gg 1$ the forces are given by $F_{\parallel} = -b$ and $F_{\perp} = -\tfrac{3}{2} b$, hence $-b$ is the typical magnitude of the ratio of the thermomagnetic and thermal forces.

The quantity b can be expressed [19, 20] by ratios of collision integrals of the Waldmann-Snider equation, the relevant kinetic equation [21]. By the way, $-b$ is equal to the quantity A occurring in Levi, McCourt and Beenakker's [11] theory of the thermomagnetic torque. In the notation of Ref. 20, b is written as

$$b = \tfrac{3}{5}\, a_{bt}\, a_{rbt} (\lambda\, \lambda^{t})^{1/2}/\lambda^{trans}\,, \tag{2.4}$$

with

$$a_{bt} = \omega_{bt} \left(\tfrac{3}{5}\,(1 - A_{tr})\,\omega_b\,\omega_t\right)^{-1/2} \cdot \left(1 + \frac{\omega_{br}}{\omega_{bt}} \frac{(-\omega_{tr})}{\omega_r}\right),\quad A_{tr} = \frac{\omega_{tr}^2}{\omega_t\,\omega_r}\,. \tag{2.5}$$

The total heat flux has the same field dependence as its translational part. The relative changes in the heat conductivity coefficients $\lambda_{\parallel}$, $\lambda_{\perp}$ and $\lambda_{trans\,v}$ are obtained [19, 20] from Eq. (2.3) if b is replaced by $\tfrac{3}{5}\, a_{rbt}^2$. Thus, the absolute value of a_{rbt} can be derived from the saturation values of $\lambda_{\parallel}$ and $\lambda_{\perp}$ [19]

$$|a_{rbt}| = \left(\frac{10}{9}\left(\frac{\lambda - \lambda_{\perp}}{\lambda}\right)_{sat}\right)^{1/2} = \left(\frac{5}{3}\left(\frac{\lambda - \lambda_{\parallel}}{\lambda}\right)_{sat}\right)^{1/2}. \tag{2.6}$$

The H/p_0 values where half of the saturation is reached are used to obtain the relaxation constant ω_b [6].

The coefficients ω_t, ω_r characterize the relaxation of the translational and rotational heat fluxes, respectively, and ω_{tr} determines their mutual coupling. Values for ω_t, ω_r and ω_{tr} can be derived from data on the shear viscosity η, the bulk viscosity η_V and the heat conductivity λ (see e. g. Ref. 20). Then the ratios

$$\frac{\lambda}{\lambda^{t}} = 1 + 2\left(-\frac{\omega_{tr}}{\omega_r}\right) + \frac{\omega_t}{\omega_r}\,,\quad \frac{\lambda^{trans}}{\lambda^{t}} = 1 + \left(\frac{-\omega_{tr}}{\omega_r}\right)$$

can be calculated. However, the translational heat conductivity λ^{trans} can be obtained more easily from [11]

$$\lambda^{trans}/\lambda = \left(\frac{15}{4}\frac{k\,\eta}{m\,\lambda} + \frac{5}{6}\, c_r\, c_t\, \eta/\eta_v\right)\left(1 + \frac{5}{6}\, c_r\, \eta/\eta_v\right)^{-1},$$

where $c_r = c_{rot}/(c_{rot} + \tfrac{3}{2} k)$, $c_t = 1 - c_r$, with c_{rot} rotational heat capacity per molecule.

Finally, the relaxation constants ω_{bt}, ω_{br} describe the collisional coupling of the Kagan vector with the translational and rotational heat fluxes. The absolute value of ω_{bt} is determined by data on the Senftleben-Beenakker effect of viscosity via the relation $\omega_{bt} = \omega_{\eta T}/\sqrt{5}$, with [22]

$$|\omega_{\eta T}| = \left(\omega_{\eta}\, \omega_T \left(\frac{\eta - \eta_3}{\eta}\right)_{sat}\right)^{1/2}.$$

Here ω_{η} and ω_T pertain to the relaxation of the frictional pressure tensor and of the tensor polarization $\mathbf{a} \propto \langle\overline{\boldsymbol{J}\boldsymbol{J}}\rangle$, respectively, and $\omega_{\eta T}$ is a measure of their collisional coupling. Values for ω_{η} are obtained from the field free shear viscosity η, and numbers for ω_T are derived from the characteristic H/p_0 values of the saturation-type curves of the field dependent viscosity coefficients [6, 22], e. g. of η_3.

The quantity ω_{br} can now be calculated [20]. But, since the signs s_t of ω_{bt} (and $\omega_{\eta T}$) and s_{rt} of a_{rbt} are not yet known, there are two possible values for $|\omega_{br}|$, according to the two possibilities $s_t = \pm s_{rt}$. In general, the sign of ω_{br} is unknown; in particular for the gases [20] HD, CH_4, CO and N_2 it is equal to that of a_{rbt}. Then, there are two possible values for the quantity b, and consequently there are two possible theoretical values for the thermomagnetic force. This same problem occurs in the theories of the thermomagnetic torque [11] and of the thermomagnetic pressure difference [12]. Comparison with experimental data weakly indicates that $s_t = s_{rt}$ applies. Notice that the cross sections $\sigma\binom{1010}{1200}$ and $\sigma\binom{1001}{1200}$ used by Levi, McCourt and Beenakker [11] are related to σ_{bt} and σ_{br} (as derived from ω_{bt} and ω_{br}) through $\sigma_{bt} = -\sigma\binom{1010}{1200}$, $\sigma_{br} = (5\,k/2\,c_{rot})^{1/2}\, \sigma\binom{1001}{1200}$. The measurement of the thermomagnetic force in the Waldmann limit gives a more direct possibility to determine whether $s_t = s_{rt}$ or $s_t = -s_{rt}$ applies. By the way, measurements of the flow birefringence by Baas[23] showed that $\omega_{\eta T}$ is positive for some linear molecules (like HD, CO, N_2) i. e. $s_t = +1$.

For a calculation of b one has to know η_V, η, λ as well as the field effects on η and λ. All these data are available only for the gases HD, CH_4, CO and N_2; some quantities relevant for the thermomagnetic force are listed in Table 1. The simplifying assumption $\omega_{tr} = 0$ made by Hess[13] gives for b the value

$$b_H = s_t\, s_{rt}\; \frac{1}{5} \left[\frac{2\, m\, \lambda}{k\, \eta}\, \frac{\omega_T}{\omega_b} \left(\frac{\lambda - \lambda_{\|}}{\lambda} \right)_{sat} \left(\frac{\eta - \eta_3}{\eta} \right)_{sat} \right]^{1/2}; \tag{2.7}$$

Table 1 shows that the error introduced in this way is at least 20%.

Table 1. The quantities λ^{trans}/λ, b and b/b_H calculated for the gases HD, CH_4, CO and N_2 from the relaxation constants given in Ref. 20 and from ω_r (ω_r has been given incorrectly in Ref. 20). The signs of ω_{bt} (and $\omega_{\eta T}$) and a_{rbt} are denoted by s_t and s_{rt}, respectively.

	λ^{trans}/λ	$b * 10^3$		$b\ /\ b_H$		ω_r/ω_η
		$s_t = s_{rt}$	$s_t = -s_{rt}$	$s_t = s_{rt}$	$s_t = -s_{rt}$	
HD	0.70	0.5	−0.2	1.2	0.6	1.69
CH_4	0.60	0.9	−0.1	1.4	0.2	1.20
CO	0.71	4.5	0.8	1.7	−0.3	2.37
N_2	0.71	3.6	0.7	1.8	−0.3	2.23

3. The Thermomagnetic Force on Spheroids

According to Eqs. (1.3), (2.1) and (2.2) the total force exerted on a spheroidal particle

$$\mathbf{K}(\boldsymbol{H}) = \mathbf{K}_0 + \mathbf{K}_H \tag{3.1}$$

consists of the zero field thermal force

$$\mathbf{K}_0 = \mathbf{K}(\boldsymbol{H} = \boldsymbol{o}) = [K^{\|}\, \boldsymbol{s}\,\boldsymbol{s} + K^{\perp}(\boldsymbol{\delta} - \boldsymbol{s}\,\boldsymbol{s})] \cdot \boldsymbol{g}, \tag{3.2}$$

$$K^{\|} = \frac{2}{5}\, \frac{F}{\pi}\, \frac{\lambda^{trans}}{\bar{c}}\, |\nabla T|\, 2(1-N), \quad K^{\perp} = \frac{2}{5}\, \frac{F}{\pi}\, \frac{\lambda^{trans}}{\bar{c}}\, |\nabla T|\, (1+N), \tag{3.3}$$

and of the field dependent thermomagnetic force

$$\mathbf{K}_H = [K^{\|}\, \boldsymbol{s}\,\boldsymbol{s} + K^{\perp}(\boldsymbol{\delta} - \boldsymbol{s}\,\boldsymbol{s})] \cdot [F_{\|}\, \boldsymbol{h}\,\boldsymbol{h}\cdot\boldsymbol{g} + F_{\perp}(\boldsymbol{\delta} - \boldsymbol{h}\,\boldsymbol{h})\cdot\boldsymbol{g} + F_{tr}\, \boldsymbol{h} \times \boldsymbol{g}]; \tag{3.4}$$

$\boldsymbol{g}$ is the unit vector in the direction of $-\nabla T$.

In the approximation used in Eq. (2.3) $F_{\|}$ and $F_{\perp}$ are even functions of $\varphi_b \propto \gamma H/p_0$, and have negative values if b is positive. On the other hand, F_{tr} is an odd function of φ_b, hence its sign depends on that of the gyromagnetic ratio γ (and on the sign of b).

For a sphere, Waldmann's formula for the thermal force[15] is obtained from Eqs. (1.4) and (3.3)

$$K^{\|} = K^{\perp} = \tfrac{8}{15}\, a^2 (\lambda^{trans}/\bar{c})\, |\nabla T|, \tag{3.5}$$

i.e. $\mathbf{K}_0$ is parallel to $\boldsymbol{g}$ (and $-\nabla T$). Then, the thermomagnetic force can be simply) expressed in terms of the thermal force $\mathbf{K}_0$, giving Hess' result[13]

$$\mathbf{K}_H = F_{\|}\, \boldsymbol{h}\,\boldsymbol{h}\cdot\mathbf{K}_0 + F_{\perp}(\boldsymbol{\delta} - \boldsymbol{h}\,\boldsymbol{h})\cdot\mathbf{K}_0 + F_{tr}\, \boldsymbol{h} \times \mathbf{K}_0 . \tag{3.6}$$

If the magnetic field is parallel to the temperature gradient ($\boldsymbol{h} = \pm \boldsymbol{g}$), the thermomagnetic force is parallel to the thermal force and is determined by $F_{\|}$ only. Inspection of Eq. (3.4) shows that this is true for general spheroids. The functions $F_{\perp}$ and F_{tr} play a role if $\boldsymbol{h}$ is perpendicular to $\boldsymbol{g}$. Before discussing this case in detail, we note the expressions for $K^{\|}$ and $K^{\perp}$ as derived from Eqs. (1.5) – (1.7), for a thin disk

$$K^{\|} = \tfrac{8}{5}\, a_{\perp}^2\, (\lambda^{trans}/\bar{c})\, |\nabla T|, \quad K^{\perp} = \tfrac{1}{2} K^{\|}, \tag{3.7}$$

a long rod

$$K^{\|} = \tfrac{2}{5}\pi\, a_{\|}\, a_{\perp} (\lambda^{trans}/\bar{c})\, |\nabla T|, \quad K^{\perp} = \tfrac{3}{2} K^{\|}, \tag{3.8}$$

and a circular cylinder

$$K^{\|} = \tfrac{4}{5}\, L\, a(1 + 2\, a/L)\, (\lambda^{trans}/\bar{c})\, |\nabla T|,$$
$$K^{\perp} = \tfrac{6}{5}\, L\, a(1 + \tfrac{2}{3}\, a/L)\, (\lambda^{trans}/\bar{c})\, |\nabla T|. \tag{3.9}$$

4. Discussion of Special Orientations

For nonspherical particles the thermal force depends on the orientation of the particle's axis $\boldsymbol{s}$ relative to the temperature gradient (i.e. to $-\boldsymbol{g}$), and the thermomagnetic force depends on the relative mutual orientations of the unit vectors $\boldsymbol{s}$, $\boldsymbol{g}$ and $\boldsymbol{h}$. Some special cases are discussed in this section.

a) Magnetic field is parallel to the temperature gradient ($\boldsymbol{h} = \pm \boldsymbol{g}$)

For $\boldsymbol{h} = \pm \boldsymbol{g}$ Ep. (3.4) reduces to

$$\mathbf{K}_H = F_{\|}\, \mathbf{K}_0 . \tag{4.1}$$

and $\mathbf{K}_0$ depends on the angle between $\boldsymbol{s}$ and $\boldsymbol{g}$ [14]. In the approximation of Eq. (2.3) the function $F_{\parallel}$ can be expressed by the change $\Delta\lambda_{\parallel} = \lambda_{\parallel} - \lambda$ in the heat conductivity coefficient $\lambda_{\parallel}$:

$$F_{\parallel} = -b\,\Delta\lambda_{\parallel}/(\Delta\lambda_{\parallel})_{\mathrm{sat}}\,. \qquad (4.2)$$

All measurements of the thermomagnetic force on disks [1–5] have been performed for $\boldsymbol{s} = \boldsymbol{g}$ and $\boldsymbol{h} = \pm \boldsymbol{g}$. A decrease of the total force by application of a magnetic field has been observed for all gases corresponding to positive values of b. In the most recent experiments of Davis [5] the thermomagnetic force given by Eqs. (3.2), (3.7), (4.1) represents a close upper limit of the observed values, for the case that $s_{\mathrm{t}} = s_{\mathrm{rt}}$.

b) Magnetic field is perpendicular to the temperature gradient ($\boldsymbol{h}\cdot\boldsymbol{g} = 0$)

In this case, both functions $F_{\perp}$ and F_{tr} determine the components of the thermomagnetic force

$$\mathbf{K}_{\mathrm{H}} = F_{\perp}\,\mathbf{K}_0 + F_{\mathrm{tr}}[K^{\parallel}\,\boldsymbol{s}\,\boldsymbol{s} + K^{\perp}(\boldsymbol{\delta} - \boldsymbol{s}\,\boldsymbol{s})]\cdot\boldsymbol{h}\times\boldsymbol{g}\,, \qquad (4.3)$$

the result depending on the orientation of the particle.

If the axis $\boldsymbol{s}$ is parallel to the temperature gradient ($\boldsymbol{s} = \pm\boldsymbol{g}$) one has $\mathbf{K}_0 = K^{\parallel}\,\boldsymbol{g}$ and

$$\mathbf{K}_{\mathrm{H}} = F_{\perp}\,K^{\parallel}\,\boldsymbol{g} + F_{\mathrm{tr}}\,K^{\perp}\,\boldsymbol{h}\times\boldsymbol{g}\,. \qquad (4.4)$$

The component of $\mathbf{K}_{\mathrm{H}}$ parallel to the thermal force $\mathbf{K}_0$ is given by $F_{\perp}$ and $K^{\parallel}$ only, F_{tr} and $K^{\perp}$ determine the component perpendicular to $\boldsymbol{h}$ and $\mathbf{K}_0$.

For an orientation of the particle's axis $\boldsymbol{s}$ parallel to the magnetic field ($\boldsymbol{s} = \pm\boldsymbol{h}, \boldsymbol{s}\cdot\boldsymbol{g} = 0$) the thermomagnetic force can be simply expressed by the thermal force $\mathbf{K}_0 = K^{\perp}\,\boldsymbol{g}$:

$$\mathbf{K}_{\mathrm{H}} = K^{\perp}(F_{\perp}\,\boldsymbol{g} + F_{\mathrm{tr}}\,\boldsymbol{h}\times\boldsymbol{g}) = F_{\perp}\,\mathbf{K}_0 + F_{\mathrm{tr}}\,\boldsymbol{h}\times\mathbf{K}_0\,. \qquad (4.5)$$

Again, the component of $\mathbf{K}_{\mathrm{H}}$ parallel to $\mathbf{K}_0$ is given by $F_{\perp}$, in contradistinction to Eq. (4.4) now only $K^{\perp}$ occurs.

Finally, we consider the case that the symmetry axis $\boldsymbol{s}$ is perpendicular to both the temperature gradient and the magnetic field, $\boldsymbol{s} = \pm\boldsymbol{h}\times\boldsymbol{g}$. Then again $\mathbf{K}_0 = K^{\perp}\,\boldsymbol{g}$ applies, but

$$\mathbf{K}_{\mathrm{H}} = F_{\perp}\,K^{\perp}\,\boldsymbol{g} + F_{\mathrm{tr}}\,K^{\parallel}\,\boldsymbol{h}\times\boldsymbol{g}\,, \qquad (4.6)$$

i. e. as compared to Eq. (4.4), $K^{\parallel}$ and $K^{\perp}$ have changed places.

If, for more general orientations, the scalar products $\boldsymbol{s}\cdot\boldsymbol{g}$ and $\boldsymbol{s}\cdot(\boldsymbol{h}\times\boldsymbol{g})$ are both non zero, F_{tr} occurs in the component of $\mathbf{K}_{\mathrm{H}}$ which is parallel to the thermal force.

The thermal force reaches its highest values for such pressures where the mean free path l is comparable to the size of the particle. Then, the pressure independent Waldmann limit of the thermal force gives a good estimate of its actual value. Since the thermomagnetic force is a very small effect (less than 1% of the thermal force for the gases of Table 1) this pressure region seems to be most promising for an experimental investigation of the thermomagnetic force.

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